Statistics

Spring 2023

Lecture 15

Feb 19-8:47 AM

Testing one population standard deviation σ:

SG 27

$H_0: \sigma = \sigma_0$ | $H_0: \sigma \le \sigma_0$ | $H_0: \sigma \ge \sigma_0$

$H_1: \sigma \ne \sigma_0$ | $H_1: \sigma > \sigma_0$ | $H_1: \sigma < \sigma_0$

TTT | RTT | LTT

Always identify the claim

P-Value Method

CTS $\chi^2 = \dfrac{(n-1) \cdot s^2}{\sigma^2}$

For P-Value

$\chi^2cdf(L, U, df)$

$df = n-1$

If P-Value $> \alpha$

H_0 Valid, H_1 invalid

If P-Value $\le \alpha$

H_0 invalid, H_1 Valid

If doing TTT, find the area on each side of CTS, multiply the smaller area by 2.

Final Conclusion:

Reject the claim OR FTR the claim

May 23-6:50 PM

Given $\boxed{n=10}$, $\boxed{S=13}$, $\boxed{H_0: \sigma \le 10}$, $\boxed{\text{Claim is } H_1}$

$\boxed{\alpha=.02}$

Test the claim.

$H_0: \sigma \le 10$

$H_1: \sigma > 10$ claim, RTT

CTS

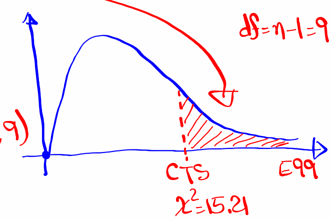
$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(10-1)\cdot 13^2}{10^2} = \boxed{15.21}$$

$df = n-1 = 9$

P_value = Area

$= \chi^2cdf(15.21, E99, 9)$

$= \boxed{.085}$



P-Value $>$ α
.085 .02

H_0 valid, H_1 invalid

Invalid claim

Reject the claim

If we choose $\alpha = .1$,
then P-Value $\le \alpha$

H_0 invalid, H_1 Valid → Valid claim

→ FTR the clai

May 23-6:57 PM

Given: $n=12$, $S=15$, $\boxed{H_0: \sigma \le 10}$, $\boxed{\text{claim is } H_0}$

$\boxed{\text{Test the claim.}}$ → No α → Use .05

$H_0: \sigma \le 10$ claim

$H_1: \sigma > 10$ RTT

CTS

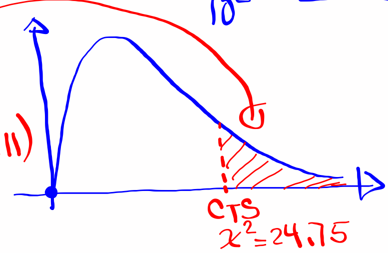
$$\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(12-1)\cdot 15^2}{10^2} = \boxed{24.75}$$

$df = n-1$
$= 11$

P_value = Area

$= \chi^2cdf(24.75, E99, 11)$

$= \boxed{.010}$



P-value $\le$ α
.010 .05

H_0 invalid, H_1 Valid

Invalid claim

Reject the claim

May 23-7:05 PM

College claims that standard deviation of ages of all students is below 8 Yrs.

No = sign → H_1 $\sigma < 8$ claim

I took a sample of 15 students, standard deviation of their ages was 6.5 Yrs.

$n = 15$
$S = 6.5$

Use $\boxed{\alpha = .1}$ to test the claim.

$H_0: \sigma \geq 8$

$H_1: \sigma < 8$ claim, LTT

CTS

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(15-1) \cdot 6.5^2}{8^2} = \boxed{9.242}$$

$df = n-1$
$= 14$

Area = P-Value = χ^2cdf$(0, 9.242, 14) = \boxed{.185}$

P-Value $>$ α → H_0 valid, H_1 invalid
.185 .1

Invalid claim

Reject the claim

If we choose $\alpha = .19, .20, \ldots$
then P-Value $\leq \alpha$

H_0 invalid, H_1 valid

Valid claim → FTR the claim

May 23-7:12 PM

The math department claims that standard deviation of scores of all exams is 10.

$\sigma = 10$ claim
↑ H_0

I took a sample of 8 exams, and standard deviation of their scores was 12.5

$n = 8$
$S = 12.5$

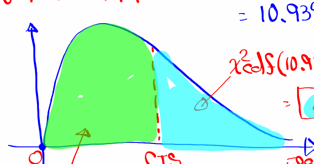
Use $\boxed{\alpha = .01}$ to test the claim.

$H_0: \sigma = 10$ claim

$H_1: \sigma \neq 10$ TTT

CTS $\overbrace{(n-1)}^{df}$

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(8-1) \cdot 12.5^2}{10^2} = 10.938$$



χ^2cdf$(10.938, E99, 7) = \boxed{.141}$

χ^2cdf$(0, 10.938, 7) = \boxed{.859}$

P-Value = 2 * Smaller area
$= 2(.141) = \boxed{.282}$

P-Value $>$ α → H_0 valid, H_1 invalid
.282 .01

Valid claim

SG 27 ✓

FTR the claim

May 23-7:22 PM

Comparing two Population Standard deviations: (SG 31)

$H_0: \sigma_1 = \sigma_2$	$H_0: \sigma_1 \geq \sigma_2$	$H_0: \sigma_1 \leq \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$	$H_1: \sigma_1 < \sigma_2$	$H_1: \sigma_1 > \sigma_2$
TTT	LTT	RTT

Always identify the claim.

1) Set-up a table

Sample 1	Sample 2
$n_1 =$	$n_2 =$
$S_1 =$	S_2

$S_1 > S_2$

$Ndf = n_1 - 1$

$Ddf = n_2 - 1$

CTS F

P-Value P

$\boxed{\text{2-SampFTest}}$

CTS $F = \frac{S_1^2}{S_2^2}$

we use Fcdf

Fcdf(L, U, Ndf, Ddf)

It is Similar to χ^2 - Dist.

Proceed with testing chart

Final conclusion

Reject the claim OR FTR the claim

May 23-7:43 PM

Use the chart below to test the claim that $\sigma_1 = \sigma_2$.

No $\alpha \to .05$

Sample 1	Sample 2
$n_1 = 8$	$n_2 = 10$
$S_1 = 12$	$S_2 = 5$

1) Verify $S_1 > S_2$ ✓

2) $Ndf = n_1 - 1 = 7$

$Ddf = n_2 - 1 = 9$

3) CTS $F = \frac{S_1^2}{S_2^2} = \frac{12^2}{5^2} = \boxed{5.76}$ ✓

4) $H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

CTS $F = 5.76$

P-Value $P = .018$ ✓

STAT TESTS

2-SampFTest

inpt: Stats

$S_1 = 12$

$n_1 = 8$

$S_2 = 5$

$n_2 = 10$

$\sigma_1 \neq \sigma_2$

P-Value $\leq \alpha$

.018 .05

H_0 invalid H_1 Valid

Invalid claim

Reject the claim

May 23-7:52 PM

Males $n=10$ $\bar{x}=30$ $S=8$

Females $n=10$ $\bar{x}=28$ $S=12$

Females	Males
$n_1=10$	$n_2=10$
$S_1=12$	$S_2=8$

$S_1 > S_2$

CTS $F=2.25$

P-Value $P=.243$ ✓

2-Samp F Test

Use $\boxed{\alpha=.1}$ to test the claim that there is a difference between two pop. stand. devs.

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$ claim, TTT

P-value > α

.243 .1

H_0 valid H_1 invalid

Invalid claim

Reject the claim

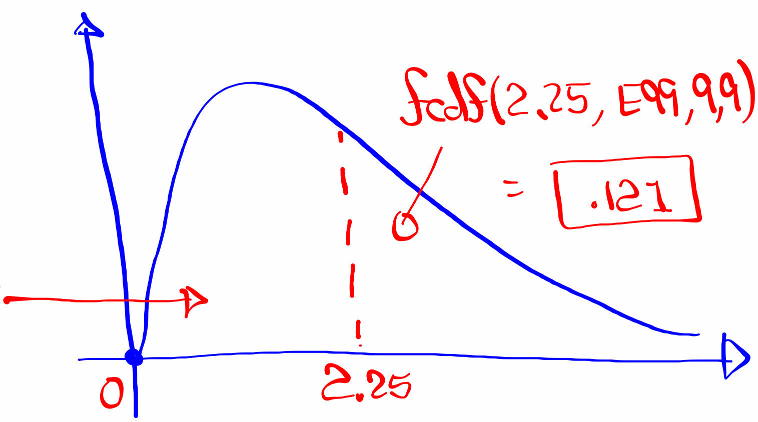
May 23-8:04 PM

CTS $F=2.25$ NDF=9, DDF=9 TTT

find P-value.

fcdf(0,2.25,9,9)

$= \boxed{.879}$



P-Value = 2 * Smaller area

$= 2(.121) = \boxed{.242}$

May 23-8:13 PM

Use the chart below to test the claim that $\boxed{\sigma_1 > \sigma_2}$.

No $\alpha \to .05$

Verify $S_1 > S_2$ ✓

Sample 1	Sample 2
$n_1 = 8$	$n_2 = 10$
$S_1 = 15$	$S_2 = 5$

$H_0: \sigma_1 \le \sigma_2$

$H_1: \sigma_1 > \sigma_2$ claim, RTT

$$\boxed{\begin{array}{l} \text{CTS } F = 9 \\ \text{P-Value } P = .002 \end{array}}$$ ✓

2-SampFTest

P-Value $\le \alpha$

.002 .05

H_0 invalid H_1 Valid

Valid claim

FTR the claim

May 23-8:16 PM

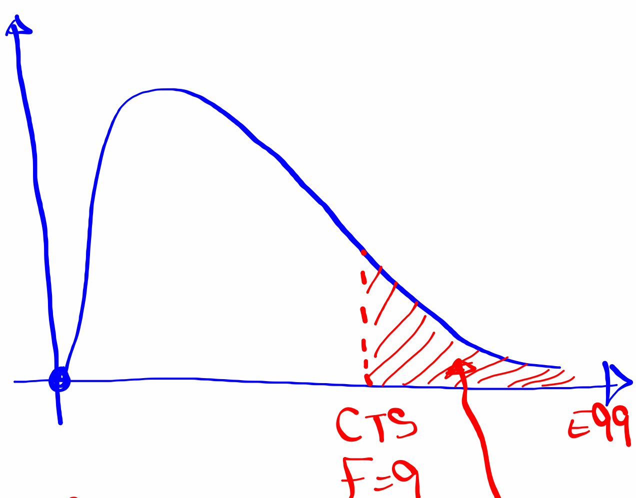
Ndf = 7

Ddf = 9

CTS F = 9

RTT

Find P-Value



P-value = Area

= fcdf(9, E99, 7, 9)

= $\boxed{.002}$

SG 31 ✓

May 23-8:22 PM

SG 35

When Comparing at least 3 Pop. means

$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_K$

H_1: At least one mean is different. RTT

K → # of Samples

n → Total Sample Size

⇒ Ndf K−1

Ddf = n−K

Method ⇒ ANOVA ⇒ Analysis of Variance

CTS F

P-Value P

⇒ STAT TESTS

ANOVA(L1, L2, L3, ...

Use P-Value method

May 23-8:35 PM

ELAC			Mt. SAC		Chaffey	
24	32	20	19	29	20	28
18	25	30	30	34	42	30
	45			38	18	20

K = 3

n = 7 + 5 + 6 = 18

⇒ Ndf = K−1 = 2

Ddf = n−K = 15

NO α → .05

Test the claim that all means are equal.

$H_0: \mu_1 = \mu_2 = \mu_3$ claim

H_1: At least one mean is different. RTT

CTS F = .250

P-Value P = .782

ANOVA(L1, L2, L3)

clear all lists

ELAC → L1

Mt.SAC → L2

Chaffey → L3

P-Value > α

.782 .05

H_0 Valid

H_1 invalid

→ Valid claim

FTR the claim

May 23-8:40 PM

I randomly selected exams from 4 different classes. Here are the Scores:

Morning	Afternoon	Evening	Online
75 83 90	72 88	70 85	86 95 100
80 100 85	65 95	98 75	98 92 100
	99	80	

$K=4$

$n = 6+5+5+6 = 22 \Rightarrow$ $Ndf = K-1 = 3$, $Ddf = n-K = 18$

No $\alpha \to .05$

$\boxed{\text{Test the claim}}$ that not all means are the Same.

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1: At least one mean is different. RTT, claim

Clear all lists, Morning $\to$ L1, Afternoon $\to$ L2, Evening $\to$ L3, Online $\to$ L4

$\boxed{\begin{array}{l}\text{CTS } F = 1.975 \\ \text{P-value } P = .154\end{array}}$

ANOVA(L1, L2, L3, L4)

P-value $> \alpha$

.154   .05

H_0 Valid H_1 invalid

invalid claim

Reject the claim

May 23-8:51 PM

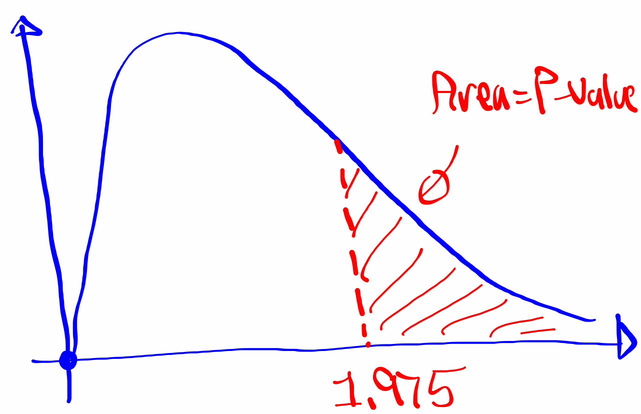
CTS $F = 1.975$

RTT

$Ndf = 3$

$Ddf = 18$

find P-Value.



$fcdf(1.975, E99, 3, 18) = \boxed{.154}$

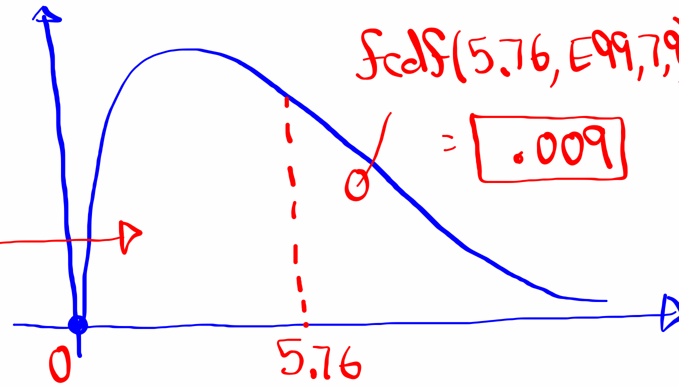
SG 35 ✓

May 23-9:03 PM

CTS $F = 5.76$, $Ndf = 7$, $Ddf = 9$, TTT

find P-Value.

$fcdf(0, 5.76, 7, 9) = \boxed{.991}$



$fcdf(5.76, E99, 7, 9) = \boxed{.009}$

P-value $= 2 *$ Smaller area

$= 2(.009) = \boxed{.018}$

May 23-8:01 PM